Temperature dependence of craze shape and fracture in poly(methyl methacrylate)

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The shape of the craze at the tip of a slowly moving crack has been determined by optical microscopy for poly(methyl methacrylate) over the range of temperatures −30° to +45°C. In all cases the craze shape can be described by the Dugdale model for the plastic zone at a crack tip. It was of particular interest that the crack opening displacement was found to be constant over the whole temperature range. Fracture toughness values deduced from the craze shape were in good agreement with those obtained directly. Quantitative estimates of the craze stress were obtained and are discussed.

INTRODUCTION

In a previous publication, a study was reported of the shape of the primary craze at the tip of a crack in two grades of poly(methyl methacrylate) (PMMA), using optical microscopy[1]. It was found that the craze shape was close to that predicted by the Dugdale model for the plastic zone at a crack tip[2]. It was therefore possible to deduce values for the crack opening displacement, the craze stress and the fracture toughness directly from the craze shape. The fracture toughness values agreed well with those obtained from direct measurement.

There appeared to be merit in extending the initial studies to a range of temperatures, because this could give some physical insight into the nature of the two parameters of the model, the crack opening displacement and the craze stress. In addition to the craze shape measurements, the fracture toughness has also been measured over the same temperature range at a single crack speed. This enables the link between the two types of measurement to be made convincingly. Since the present investigation was initiated, an account of the fracture toughness behaviour of PMMA as a function of crack speed and temperature has been reported by Williams and coworkers[3]. The work described here is complementary to such engineering studies, and the interrelationship between the two investigations will be discussed.

EXPERIMENTAL

Material

The measurements were all carried out on commercial grade PMMA manufactured by ICI Ltd, under the trade name Perspex cast sheet.

Measurement of craze shape

The craze shape measurements were made on small compact tension (CT) specimens of dimensions ~5 x 10 x 3 mm (*Figure 1*). An initial notch was introduced by making a saw cut into the centre of one edge. The crack was then initiated and propagated by pushing a razor blade slowly into the saw cut, the line of the crack being in the direction of the initial saw cut. When the specimen is viewed in an optical microscope in reflected light, looking vertically in a direction normal to the plane of the crack and craze (arrow direction in *Figure 1*), the crack and the craze both produce a series of interference fringes. In these experiments, the crack moved sufficiently slowly for the loaded moving crack and craze to be photographed. The illumination was

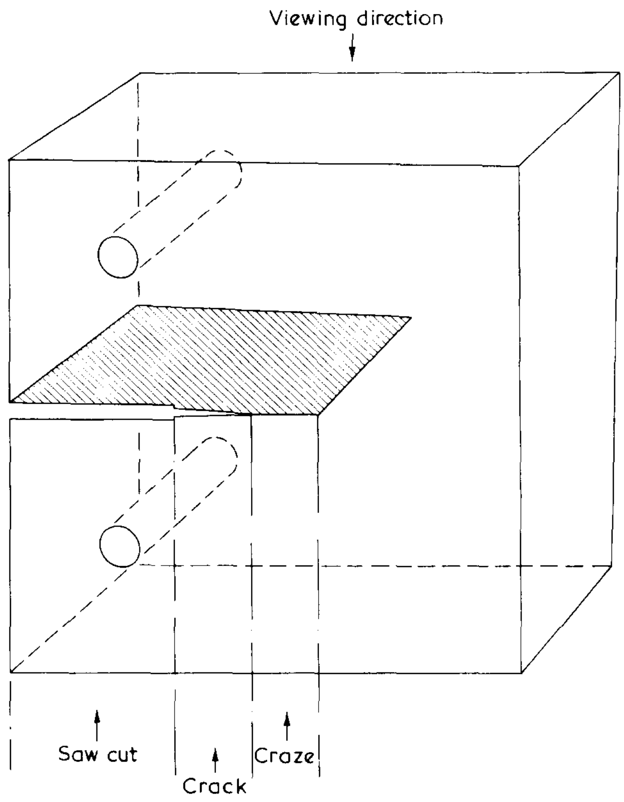

Figure 1 The compact tension specimen

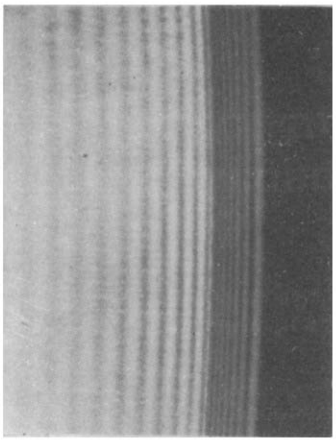

Figure 2 Photograph of a typical loaded moving crack and craze in PMMA

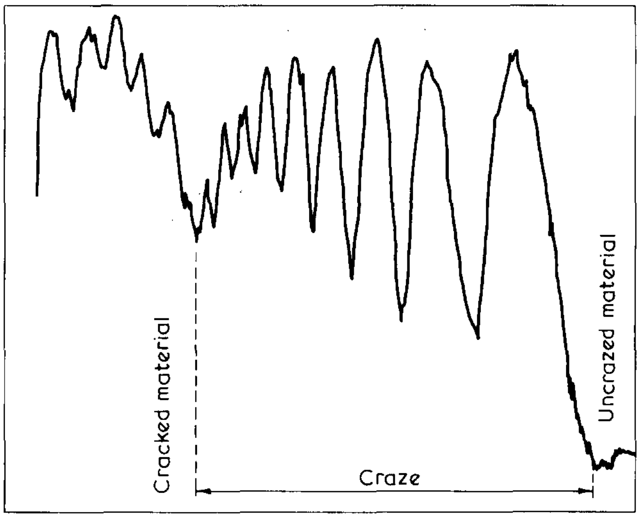

Figure 3 Typical microdensitometer trace of fringes in a PMMA crack and craze

achieved by a mercury arc lamp with a filter, so that the fringe patterns are produced by monochromatic light at 551 nm. A typical photograph is shown in *Figure 2*. A microdensitometer trace shows the position of the fringes very clearly and can therefore be used for quantitative measurements, as illustrated by *Figure 3*.

Fracture toughness measurements

Fracture toughness measurements were carried out on larger CT specimens of dimensions 50 x 46.7 mm, in accordance with the design for such specimens given by Brown and Srawley[4]. The overall specimen is similar in shape to that shown in *Figure 1*, and again the initial crack would be formed by inserting a razor blade into the saw cut. The specimen is then mounted in an Instron tensile testing machine and the crack propagated by driving apart at constant speed clamps inserted into the two dotted holes shown in the specimen. The stress intensity factors K_c were calculated following the boundary collocation calculations in ref 3, measuring the load P as the crack passed scratch lines marked on the crack path.

The compliance C of the CT specimen is given by the relationship

$$C = \frac{2}{E^*BW} \int Y^2(a)\mathrm{d}a \qquad (1)$$

where

$$Y = K_c \frac{BW^{1/2}}{P} \qquad (2)$$

In these expressions B and a are the crack width and crack length respectively, W is the specimen length and E^* the reduced modulus ($E^* = E$ in plane stress and $E/1 - \gamma^2$ in plane strain, where γ is Poisson's ratio). E^* was determined as a function of temperature from plots of the calculated values of E^*C using equation (1) *versus* the measured specimen compliance, Y/P, where Y is the grip extension for a load P.

RESULTS

Comparison of craze shape with Dugdale plastic zone model

A detailed discussion of the shape of the crazes in PMMA at room temperature in different situations (loaded stationary, loaded moving and unloaded) was given in the previous paper. Here we will only be concerned with loaded moving crazes, and the results of the previous investigation will be drawn upon where required.

The Dugdale plastic zone model considers that yielding of the material at the crack tip makes the crack longer by the length R of the plastic zone. The stress singularity at the crack tip is exactly cancelled by a series of compressive stresses of magnitude σ_0 which act on the extended crack surface i.e. the boundary of the plastic zone (see *Figure 4*).

The length of the plastic zone is related to the fracture toughness K_c by:

$$R = \frac{\pi}{8} \frac{K_c^2}{\sigma_0^2} \qquad (3)$$

and the crack opening displacement (*COD*), δ_t given by:

$$\delta_t = \frac{8\sigma_0 R}{\pi E^*} = \frac{K_c^2}{\sigma_0 E^*} \qquad (4)$$

The thickness of the plastic zone δ at any point x, is:

$$\delta = \frac{8}{\pi E^*} \sigma_0 R \left[\xi - \frac{x_1}{2R} \log \left(\frac{1+\xi}{1-\xi} \right) \right] \qquad (5)$$

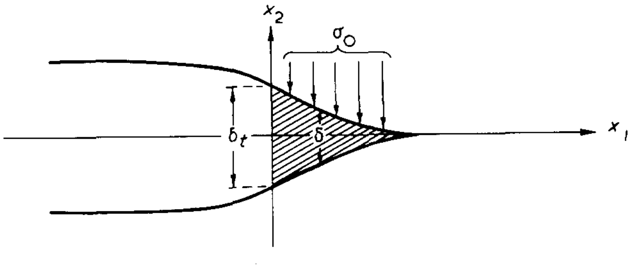

Figure 4 The Dugdale plastic zone

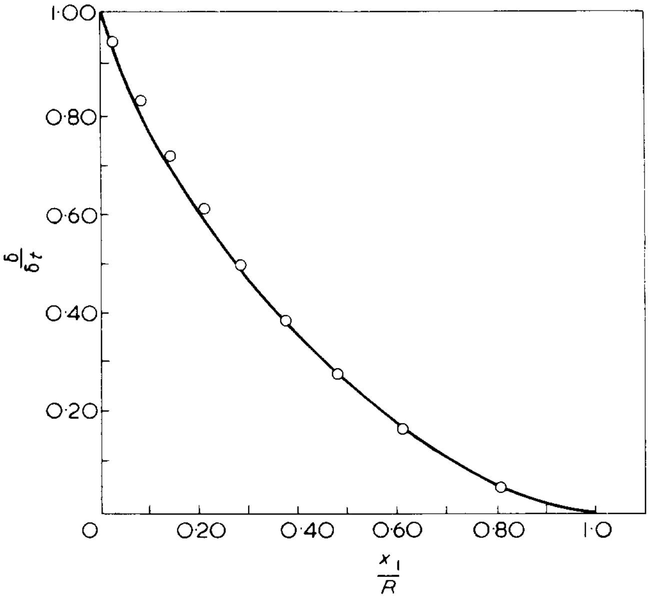

Figure 5 Fit to shape of craze zone using the Rice equation for the separation distance in the craze δ as a function of the distance from the crack tip x_1. Temperature 0°C. ○, Observed; ——, calculated

where

$$\xi = \left(\frac{1 - x_1}{R}\right)^{1/2}$$

These expressions are based on a comprehensive treatment of the problem given by Rice[5].

In the previous investigation it was shown that the retardation pattern of the loaded moving craze was in excellent agreement with equation (5). This agreement has been confirmed in the present investigation and results for the craze shapes at 0° and --20°C are shown in *Figures 5* and *6* respectively. In achieving these fits it is assumed that the refractive index of the craze is constant along its length, an assumption which receives support from the fact that the retardation pattern of the craze at break is exactly proportional to that at zero load.

To find the absolute value of the crack opening displacement δ_t requires a value for the refractive index of the craze at break μ_B. In the previous paper[1] it is shown that this is related to the refractive index of the relaxed craze μ_0 by the relationship:

$$\frac{\mu_B\lambda}{\mu_0} = \frac{n_b}{n_0} = 1 + \frac{\lambda - 1}{1.32} \qquad (6)$$

where λ is the extension ratio of the craze at break, and n_b and n_0 are the number of fringes in the craze at break and unloaded respectively. At all temperatures the number of fringes in the moving loaded craze was nine, compared with between three and four in the unloading craze. This gives a value for λ of 3.1 and a value for μ_B of 1.1, based on the value of 1.32 for μ_0 found by Kambour[6]. This assumes that the refractive index of PMMA is temperature independent. Measurements of the refractive index of PMMA and related polymers by Wiley and Brauer[7] confirm that this is a reasonable assumption. Their results show that the refractive index of PMMA increases by 0.4% when the temperature is lowered from 20° to −30°C and decreases by 0.2% when the temperature is raised from 20° to +45°C. Such changes would not be significant at the level of accuracy which can be achieved here. It is therefore concluded that the crack opening displacement is constant over the temperature range studied i.e. from −30° to +45°C, and is estimated to be 2.2 μm, on the basis of the arguments presented above.

It is instructive to use the craze shape data to calculate values for the fracture toughness K_c and the craze stress σ_0. This was carried out using equations (3) and (4) together with the experimental values of E^*. The collected results are shown in *Table 1*, and a plot of the calculated values for the fracture toughness *versus* the experimental values is shown in *Figure 7*. This plot indicates that the fracture toughness of PMMA under the experimental conditions of the microscope craze shape measurements is to a good approximation a factor 0.71 of that measured in the Instron CT tests over the temperature range from −30° to +45°C. It is considered that this constant difference between the fracture toughness values for the two tests can be attributed to the different crack speeds in the two tests. Good supporting evidence for this conclusion comes from the data of Williams and coworkers[3]. They report an initiation value for K_c of about 0.7 MN/m^2 at 20°C, which is in excellent agreement with that deduced here from the craze shape measurements. Williams and coworkers also showed that plots of fracture toughness K_c *versus* log crack speed for different temperatures formed a series of parallel straight lines to a good approximation. It is therefore reasonable to expect that the ratio of the fracture toughness for crack initiation to that at a crosshead speed of 5×10^{-3} cm/min would be constant over the temperature range examined here. This expectation is borne out by the results shown in *Figure 7*.

Finally it is of interest to consider the values deduced for the craze stress σ_0. First, it is to be noted that the room temperature craze stress value of 83 MN/m^2 is very close to that obtained in the previous craze shape investigations[1]. It is only slightly less than the tensile yield stress value of 100 MN/m^2 which can be deduced from measure-

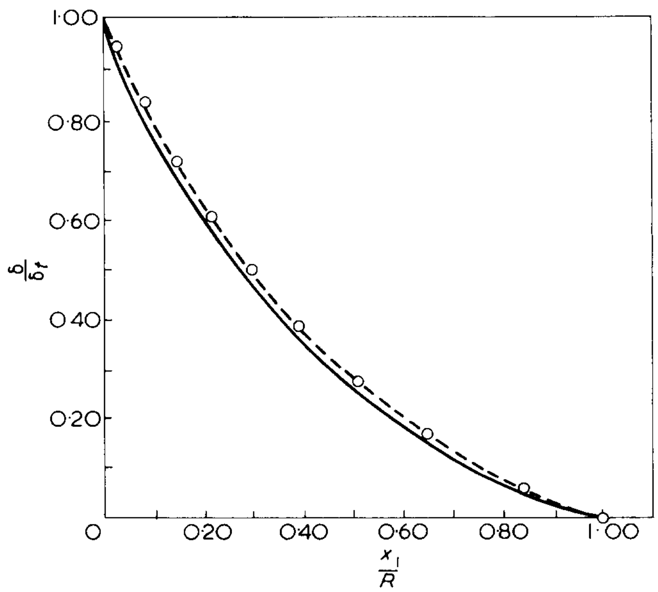

Figure 6 Craze shape fit to Rice equation at −20°C. ○, Observed, ——, calculated

Table 1 Collected data from craze shape and fracture toughness measurements

Specimen	Temperature (°C)	Measured COD δ_t(μm)	Measured craze length, R (μm)	Measured fracture toughness, K_c (MN/m$^{3/2}$)	E^* (GN/m^2)	Fracture toughness calculated from craze shape (MN/m$^{3/2}$)	Craze stress calculated from craze shape (MN/m^2)
CT 6PT 3	−30	2.2	35.0	1.28	3.93	0.92	97
CT 5PT 3	−20	2.2	34.2	1.22	3.71	0.88	94
CT 4PT	−20	2.2	33.7	1.20	3.68	0.88	94
CT 3PT	−10	2.2	32.8	1.18	3.58	0.86	94
CT 21PT	−10	2.2	33.3	1.16	3.54	0.85	92
CT 3PT 3	0	2.2	30.7	1.14	3.35	0.83	94
CT 20 PT	0	2.2	30.5	1.13	3.33	0.83	94
CT 19 PT	+10	2.2	28.9	1.08	3.01	0.77	90
CT 1PT 3	+20	2.2	29.0	1.02	2.77	0.71	83
CT 1 PT	+20	2.2	28.4	1.00	2.72	0.70	83
CT 14 PT	+20	2.2	28.7	1.04	2.85	0.73	85
CT 7PT 3	+32.5	2.2	28.6	0.84	2.33	0.61	70
CT 7PT	+35	2.2	29.0	0.87	2.34	0.60	70
CT 8PT	+45	2.2	28.9	0.75	1.96	0.50	59

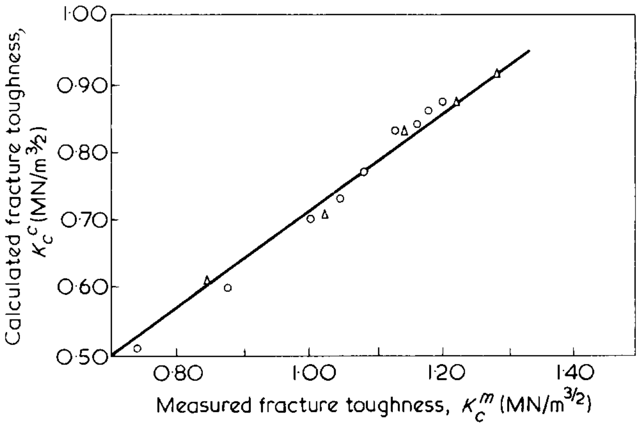

Figure 7 Plot of calculated fracture toughness K_c^c against measured fracture toughness K_c^m. Line K_c^c/K_c^m = 0.71

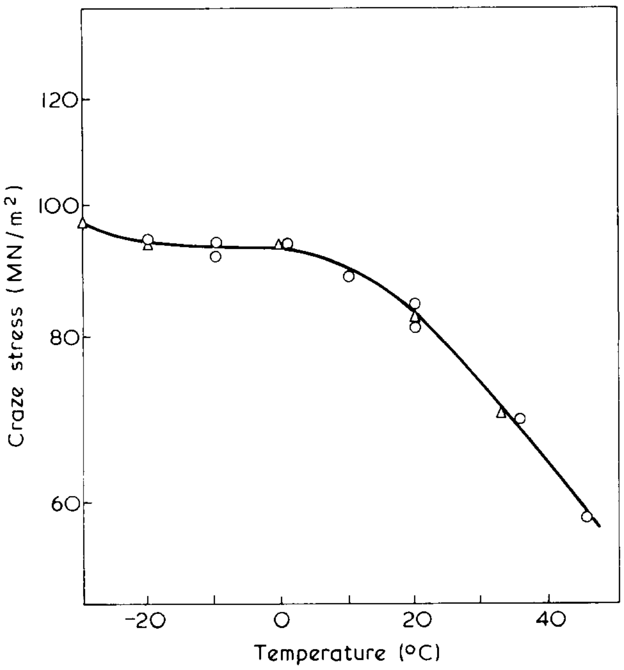

Figure 8 Calculated craze stress σ_0 as a function of temperature

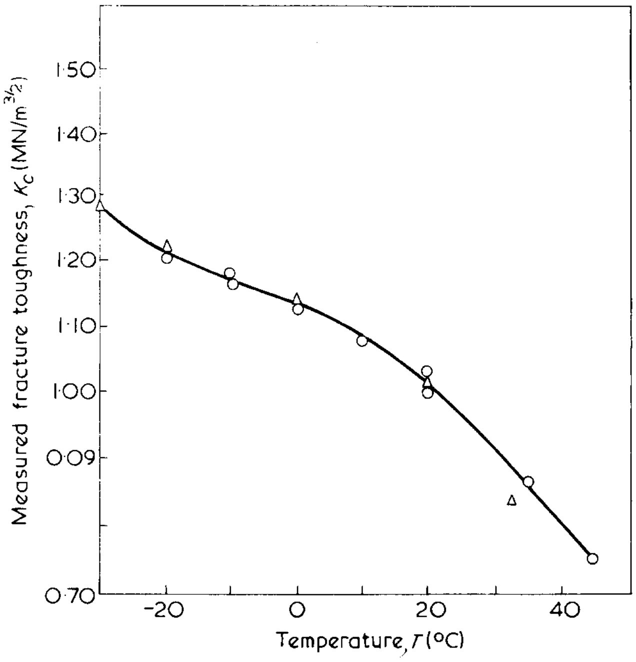

Figure 9 Measured fracture toughness K_c as a function of temperature

ments of shear yield stress at comparably low strain rates[8]. Secondly, the temperature dependence of the craze stress, shown in *Figure 8*, differs from that of the fracture toughness on modulus (*Figures 9* and *10*). In particular the craze stress is very nearly constant from −30° to 0°C whereas the modulus changes by 15%, and the fracture toughness by almost as much. Most of the change in the fracture toughness K_c below room temperature is due to changes in the modulus, whereas the changes at higher temperatures can be attributed to changes in both modulus and craze stress. It is interesting that the temperature dependence of the craze stress is not exactly analogous to the reported temperature dependence of the yield stress. In the temperature range below room temperature where the craze stress is nearly constant, the yield stress is changing rapidly, according to data collated by Bowden[9].

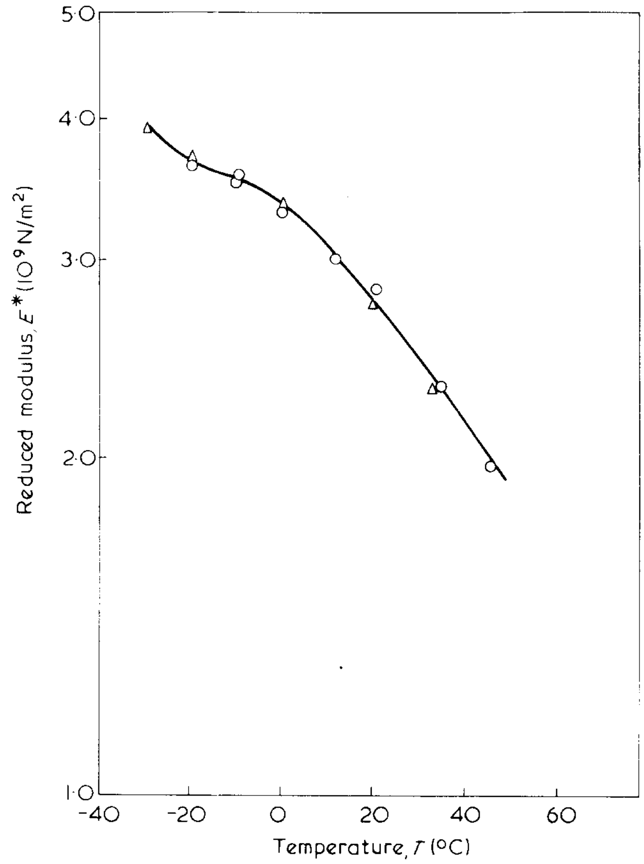

Figure 10 Reduced modulus E^* as a function of temperature

CONCLUSIONS

It has been shown the shape of the craze at the tip of a sharp crack in PMMA is in good agreement with that expected on the basis of the Dugdale plastic zone model for slow crack propagation over a wide range of temperatures. The results show that the crack opening displacement is constant to a very good approximation indeed. This had previously been inferred to be a realistic approximation from fracture toughness data, and now receives direct confirmation.

The changes in fracture toughness with temperature can therefore be ascribed to changes in the modulus and craze stress, and the measurements of compliance and craze stress give a direct indication of their relative importance.

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